

1 Make sense of problems and persevere in solving them.

Focus 1A: Entry into a problem

Determines the form (quantity or measure) of the solution to the mathematical or real-life problem, analyzes the givens to extract only the pertinent information needed to solve the mathematical or real-life problem, and analyzes the givens to identify missing information needed to solve the mathematical or real-life problem.

- Explain the meaning of a problem
- Look for entry points to its solution
- Analyze givens, constraints, relationships, and goals for extraneous or missing information
- Make conjectures about the form and meaning of the solution

Focus 1B: Solution path

Determines and uses an appropriate solution path including the identification and appropriate use of tools to solve a well-posed mathematical or real-life problem.

- Plan a solution pathway rather than simply jumping into a solution attempt
- Consider analogous problems
- Try special cases and simpler forms of the original problem in order to gain insight into its solution
- Monitor and evaluate progress and change course if necessary
- Transform representations to get the information they need
- Understand the approaches of others to solving complex problems and identify correspondences between different approaches

Focus 1C: Appropriate solution to a problem

Determines whether the solution to a well-posed mathematical or real-life problem is accurate and/or realistic.

- Check solutions to problems using a different method
- Ensure the solution makes sense
- Verify the necessary precision is used in the solution
- Analyze the problem to ensure the proper units are specified in the solution

2 Reason abstractly and quantitatively.

Focus 2A: Decontextualize and manipulate

Abstracts a given situation, represents it symbolically, and manipulates the symbols.

- Decontextualize—
 - To abstract a given situation
 - To represent it symbolically
 - To manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents
 - Create a coherent representation of the problem at hand

Focus 2B: Contextualize

Shows understanding of the referents for the symbols involved in a mathematical sentence representing a situation.

- Contextualize—
 - To pause as needed during the manipulation process in order to probe into the referents for the symbols involved
- Consider the units involved

Focus 2C: Quantitative reasoning

Knows and uses different properties of operations and objects and shows understanding of the meaning of the quantities.

- Make sense of quantities and their relationships in problem situations
- Attend to the meaning of quantities, not just how to compute them
- Know and flexibly use different properties of operations and objects

3 Construct viable arguments and critique the reasoning of others.

Focus 3A: Construct arguments

Makes and defends arguments.

- Understand and use stated assumptions, definitions, and previously established results in constructing arguments
- Communicate arguments properly to others
- Reason inductively about data, making plausible arguments that take into account the context from which the data arose
- Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions whereas middle school and high school students learn to determine domains to which an argument applies

Focus 3B: Evaluate arguments

Evaluates arguments.

- Respond to the arguments of others
- Compare the effectiveness of two plausible arguments
- Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is
- Listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments

4 Model with mathematics.

Focus 4A: Create models.

Creates an appropriate model.

- Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas

Focus 4B: Interpret and analyze models.

Interprets and analyzes models.

- Analyze relationships mathematically to draw conclusions
- Improve models if they have not served its purpose
- Interpret their mathematical results in the context of the situation
- Explain correspondences between equations, verbal descriptions, tables, and graphs

Focus 4C: Use the model to solve problems.

Uses a model to solve a problem.

- Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace
- Use models to develop designs, predict outcomes, describe phenomena, solve problems, and explain causes and effects
- Make assumptions and approximations to simplify a complicated situation
- Reflect on whether the results make sense
- Draw diagrams of important features and relationships, graph data, and search for regularity or trends
- Rely on using models to help conceptualize and solve a problem

7 Look for and make use of structure.

Focus 7/8A: Structure

Look for and make use of structure.

- Use the structure of an expression to rewrite it in another form
- Step back for an overview and shift perspective
- See complicated things as single objects or as being composed of several objects
- Use the geometric attributes of figures to classify/sort
- Use mathematical properties of numbers, operations, and equality to explain and analyze mathematical or real-world problems
- Recognize the elements of effective data representation for a data set
- Use familiar/known structures to see something in a different way

8 Look for and express regularity in repeated reasoning.

Focus 7/8B: Patterns

Look for and express regularity in repeated reasoning.

- Discern a pattern
- Notice if calculations result in repeated values (e.g., notice when there is a repeating decimal)
- Look both for general methods and for shortcuts
- Abstract general principles from repeated phenomena (e.g., slope, formulas for area or perimeter, correlation)